



**DERIVING EXHAUSTIVE TEST SUITES FOR  
NONDETERMINISTIC FSMs  
W.R.T. NON-SEPARABILITY RELATION**

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# OUTLINE

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- Introduction
- Relations between NFSMs and 'all weather conditions' assumption
- Separating two NFSMs
- Test suite derivation w.r.t. a mutation machine
- Test suite derivation w.r.t. "black box"
- Future work

# WHY NONDETERMINISTIC?



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- FSMs serve as an underlying model for SDL specifications as well as for UML state charts and those FSMs are nondeterministic
- PROMELA that is used in the model checker SPIN also allows a nondeterministic behavior
- software mutation testing

Nondeterministic descriptions usually are more compact than the corresponding deterministic models

# FINITE STATE MACHINE

$$\mathcal{S} = \langle S, I, O, h_S, s_1 \rangle$$

**S** – set of states

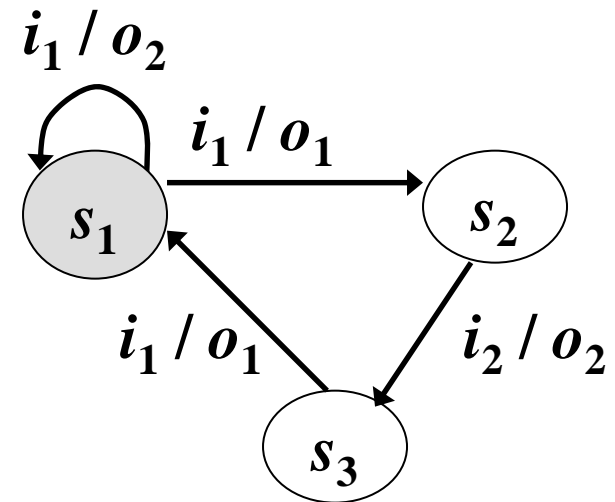
**I** – input alphabet

**O** – output alphabet

**$h_S$**  – behavior relation,

$$h_S \subseteq S \times I \times O \times S$$

**$s_1$**  – the initial state



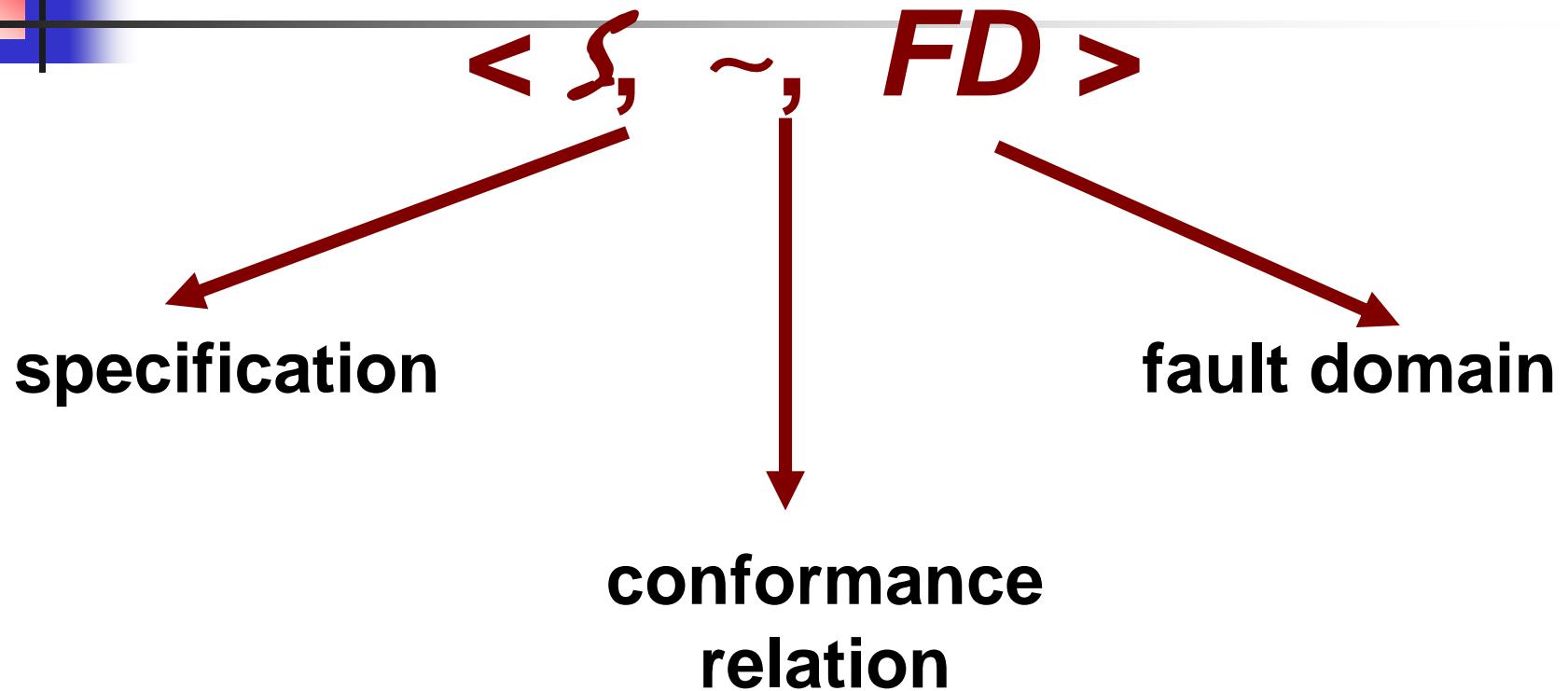
$$out_S(s, \alpha) = \{ \beta \mid \exists s' \in S (s, \alpha, \beta, s') \in h_S \}$$

# RELATIONS BETWEEN NFSMs

- FSMs  $T$  and  $S$  are *equivalent* if
$$\forall \alpha \in I^* (out_T(t_1, \alpha) = out_S(s_1, \alpha))$$
- FSM  $T$  is a *reduction* of  $S$  if
$$\forall \alpha \in I^* (out_T(t_1, \alpha) \subseteq out_S(s_1, \alpha))$$
- FSMs  $T$  and  $S$  are *non-separable* if
$$\forall \alpha \in I^* (out_T(t_1, \alpha) \cap out_S(s_1, \alpha) \neq \emptyset)$$
- FSMs  $T$  and  $S$  are *r-compatible* if  $T$  and  $S$  have a common reduction

separability  $\Leftrightarrow$  non-reduction  $\Leftrightarrow$   $r$ -distinguishability  $\Leftrightarrow$  non-equivalence

# TRADITIONAL FAULT MODEL



An implementation FSM *Imp* is *conforming* if  $Imp \sim S$

The objective of MBT testing – to detect each nonconforming  $Imp \in FD$



# 'ALL WEATHER CONDITIONS' ASSUMPTION

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holds



- + there exist test suite derivation methods
- sometimes this assumption is not realistic
- each test case is applied several times

does not hold



- + realistic
- + each test case is applied only once
- existing test derivation methods do not guarantee the fault coverage



# DETECTING NONCONFORMING IUT WHEN 'ALL WEATHER CONDITIONS' ASSUMPTION DOES NOT HOLD

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- **Preset tests (experiments)**

Guarantee to detect an implementation that is separable with the specification

- **Adaptive tests (experiments)**

Guarantee to detect an implementation that is  $r$ -distinguishable with the specification



# NON-SEPARABILITY RELATION

Complete FSMs  $T$  and  $S$  are *non-separable* if for each input sequence  $\alpha$

$$[out_T(t_0, \alpha) = out_S(s_0, \alpha)]$$

If there exists an input sequence  $\alpha$   
[ $out_T(t_0, \alpha) \neq out_S(s_0, \alpha)$ ],  
then FSMs  $T$  and  $S$  are *separable*

$\alpha$  - a *separating* sequence of FSMs  $T$  and  $S$



# TESTING W.R.T. THE NON-SEPARABILITY RELATION

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- Fault model  $\langle S, \sim, FD \rangle$  where

$S$  - the specification FSM

$\sim$  - the non-separability relation

$FD$  – the fault domain

All FSMs are complete

# EXHAUSTIVE AND SOUND TESTS

- A *test suite* - a finite set of finite input sequences
- A test suite is *exhaustive* w.r.t.  $\langle S, \sim, FD \rangle$  if the test detects each  $Imp \in FD$  that is separable with  $S$
- A test suite is *sound* w.r.t.  $\langle S, \sim, FD \rangle$  if each  $Imp \in FD$  that is non-separable with  $S$  *passes* the test suite
- A test suite is *complete* w.r.t.  $\langle S, \sim, FD \rangle$  if the test suite is sound and complete

We derive an exhaustive test suite



# FAULT MODELS

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Fault models

$\langle \mathcal{S}_i, \sim_i, \mathit{Sub}_{nd}(MM) \rangle$

and

$\langle \mathcal{S}_i, \sim_i, \mathcal{R}_m \rangle$

have finite fault domains



A test suite can be derived by **explicit enumeration**

# INTERSECTION OF FSMs

*Intersection*  $S \cap T$  is the largest connected submachine of the FSM

$\langle S \times T, I, O, h, s_1 t_1 \rangle$  where

$$(st, i, o, s' t') \in h$$



$$(s, i, o, s') \in h_S \ \& \ (t, i, o, t') \in h_T$$



# DERIVING A SEPARATING SEQUENCE OF TWO FSMs (1)

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**Input:** Complete FSMs  $S$  and  $T$

**Output:** A shortest separating sequence of FSMs  $S$  and  $T$  (if it exists)

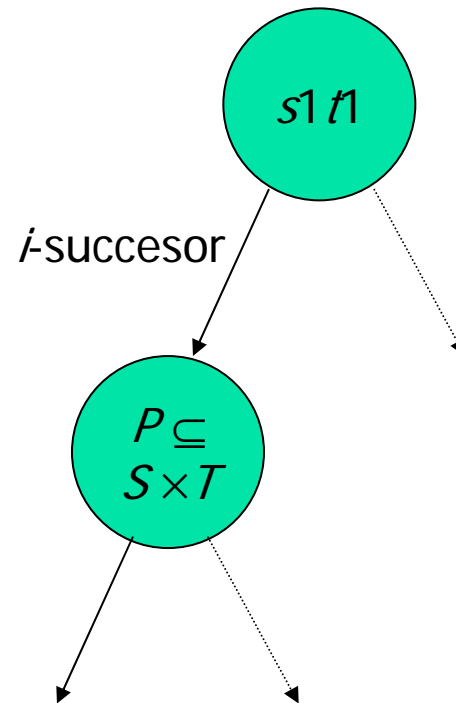
**Step 1.** Derive the intersection  $S \cap T$

If the intersection is complete then the FSMs  $S$  and  $T$  are non-separable

# DERIVING A SEPARATING SEQUENCE OF TWO FSMs (2)

**Step 2.** Derive a truncated successor tree of the intersection  $S \cap T$

Successor tree of  $S \cap T$



# TERMINATION RULES FOR A NODE WITH LABEL $P$

## Termination rule 1

There exists an input  $i$  s.t.  
for each state of the set  
 $P$  the transition under  $i$   
is undefined in the  
intersection  $S \cap T$

**successfully separated**

## Termination rule 2

There exists a node at a  
 $j$ th level,  $j < k$ , labeled  
with subset  $R \subseteq P$

**a shortest separating  
sequence cannot be  
derived using this  
path**

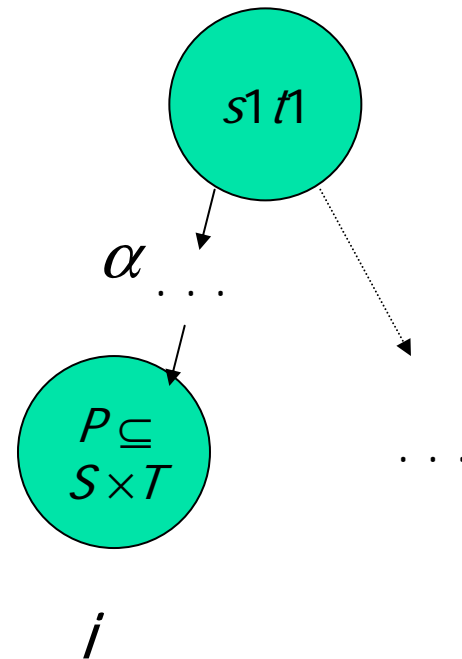


# DERIVING A SEPARATING SEQUENCE OF TWO FSMs (3)

Let there be a path labeled with  $\alpha$  to a leaf node labeled with the subset  $P$  where a transition under  $i$  is undefined for each state of  $P$

Then  $\alpha i$  is a **shortest separating sequence** of  $S$  and  $T$

Successor tree of  $S \cap T$





# UPPER BOUND ON SEPARATING SEQUENCE LENGTH

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- Given FSMs  $S$  with  $n$  states and  $T$  with  $m$  states, the length of a shortest separating sequence is at most  $2^{nm-1}$
- The upper bound is reachable **but** possibly only for exponential number of inputs
- Never experimentally reached the upper bound

# DERIVING A COMPLETE TEST SUITE

W.R.T.  $\langle S, \sim, Sub_{nd}(MM) \rangle$  (1)

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**Input:** FSMs  $S$  and  $MM$

**Output:** A complete test suite  $TS$  w.r.t.  
 $\langle S, \sim, Sub_{nd}(MM) \rangle$

**Step 1.** Derive the intersection  
 $S \cap MM$

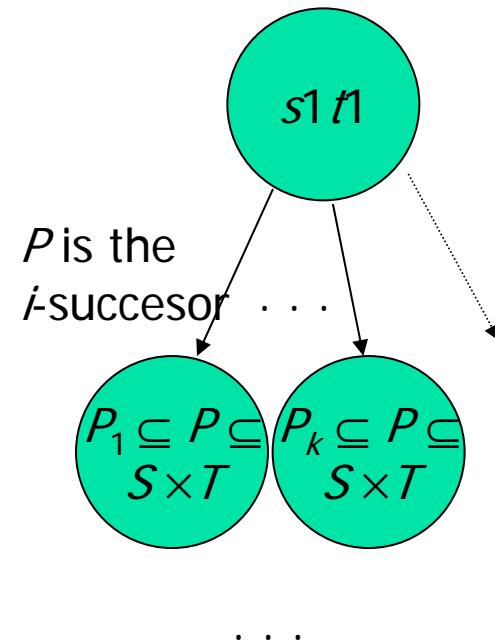
# DERIVING A COMPLETE TEST SUITE

W.R.T.  $\langle S_i, \sim_i, Sub_{nd}(MM) \rangle$  (2)

**Step 2.** Derive a truncated successor tree of the intersection  $S \cap MM$

There is an outgoing edge from a non-leaf node labeled with  $i$  to the nodes labeled with **each non-empty subset** of the  $i$ -successor of the subset  $P$

Successor tree of  $S \cap MM$



# TERMINATION RULES FOR THE NODE WITH LABEL $P$

## Termination rule 1

There exists an input  $i$  s.t.  
for each state of the set  
 $P$  the transition under  $i$   
is undefined in the  
intersection  $S \cap T$

## Termination rule 2

There exists a node of this  
path labeled with  
subset  $R \subseteq P$

## Termination rule 3

$P \supseteq \{(s, t)\}$  and the  
already derived part of  
the tree has a node  
labeled with  $\{(s, t)\}$

and

an input sequence  $\alpha$  that  
labels the path from the  
root to this node does  
not label any other path  
in the tree

# DERIVING A COMPLETE TEST SUITE

W.R.T.  $\langle S, \sim, Sub_{nd}(MM) \rangle$  (3)

- For each path terminated using Rule 1, include into  $TS$  an input sequence which labels the path appended with an input  $i$  s.t. a transition from each state of  $P$  under  $i$  is undefined in the intersection  $S \cap MM$
- For each path of the tree terminated using Rule 2 or Rule 3, include into  $TS$  the longest prefix of an input sequence that labels the path, with the following property:

For the tail input  $i$  that labels the edge from the node labeled with the set  $P$ , there exists state  $(s, m) \in P$  such that  $out_{S \cap MM}((s, m), i) \subset out_{MM}(m, i)$

# Example

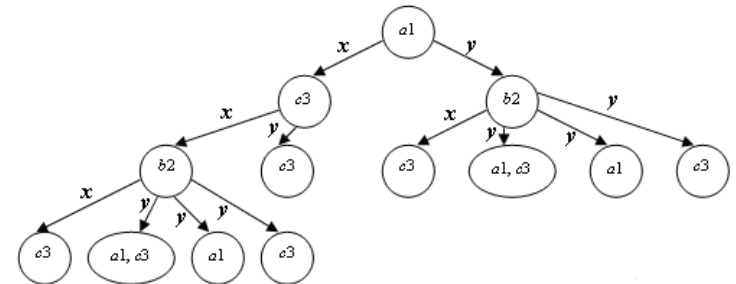
$S$	$a$	$b$	$c$	$MM$	1	2	3
$x$	$c/1$	$c/0$	$b/0$	$x$	$3/1$	$3/0$	$2/0$
$y$	$b/0$	$a/0;$ $c/1$	$c/1$	$y$	$2/0$	$1/0;$ $3/1$	$3/1;$ $1/0$

FSMs  $S$  and  $MM$

$S \cap M$	$a1$	$c3$	$b2$
$M$			
$x$	$c3/1$	$b2/0$	$c3/0$
$y$	$b2/0$	$c3/1$	$a1/0;$ $c3/1$

FSM  $S \cap MM$

Tree of the previous exhaustive test



$xy$  also is an exhaustive test suite w.r.t.  $MM$

# DERIVING A COMPLETE TEST SUITE

w.r.t.  $\langle S, \sim, \mathcal{R}_m \rangle$

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**Theorem.** Given the specification FSM  $S$  with  $n$  states a test suite which contains each input sequence of length  $2^{mn-1}$  is complete w.r.t.  $\langle S, \sim, \mathcal{R}_m \rangle$



# STATE COUNTING ALGORITHM (1)



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Main ideas:

- to derive a truncated tree of  $S$
- to terminate a path when we are sure that for each FSM  $T$  with  $m$  states the input sequence that labels the path traverses two equal subsets of the intersection  $S \cap T$

# STATE COUNTING ALGORITHM (2)



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A current node labeled with the subset  $P$  of states of  $S$  is claimed as a leaf node if the path from the root to this node has  $2^{|P| \cdot m}$  nodes labeled with subsets of  $P$



# MODIFICATION

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The node *Current* at  $k^{\text{th}}$  level labeled with subset  $P$  is a **leaf** if:

1. A subset  $K \subseteq P$  can be represented as the union of subsets  $P_j$
2. For each  $P_j$  the prefix of the path to the node at the  $l^{\text{th}}$  level,  $l < k$ , has  $(2^{|P_j| \cdot m} - 1)$  nodes labeled with the set  $P_j$
3. The suffix of the path from the node at the  $l^{\text{th}}$  level to the node *Current* has  $(2^{(|P| - |K|) \cdot m})$  nodes labeled with subsets of the set  $P$



# EXAMPLE (1)

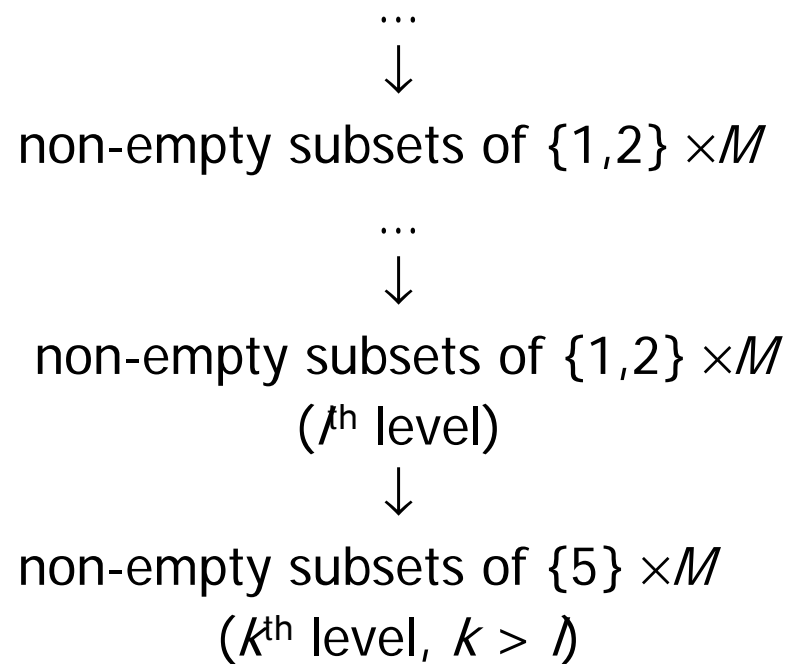
- $P = \{1,2,3,4,5\}$ ,  $M = \{a, b\}$

The number of different non-empty sets of the  $P \times M = 2^{10} - 1$

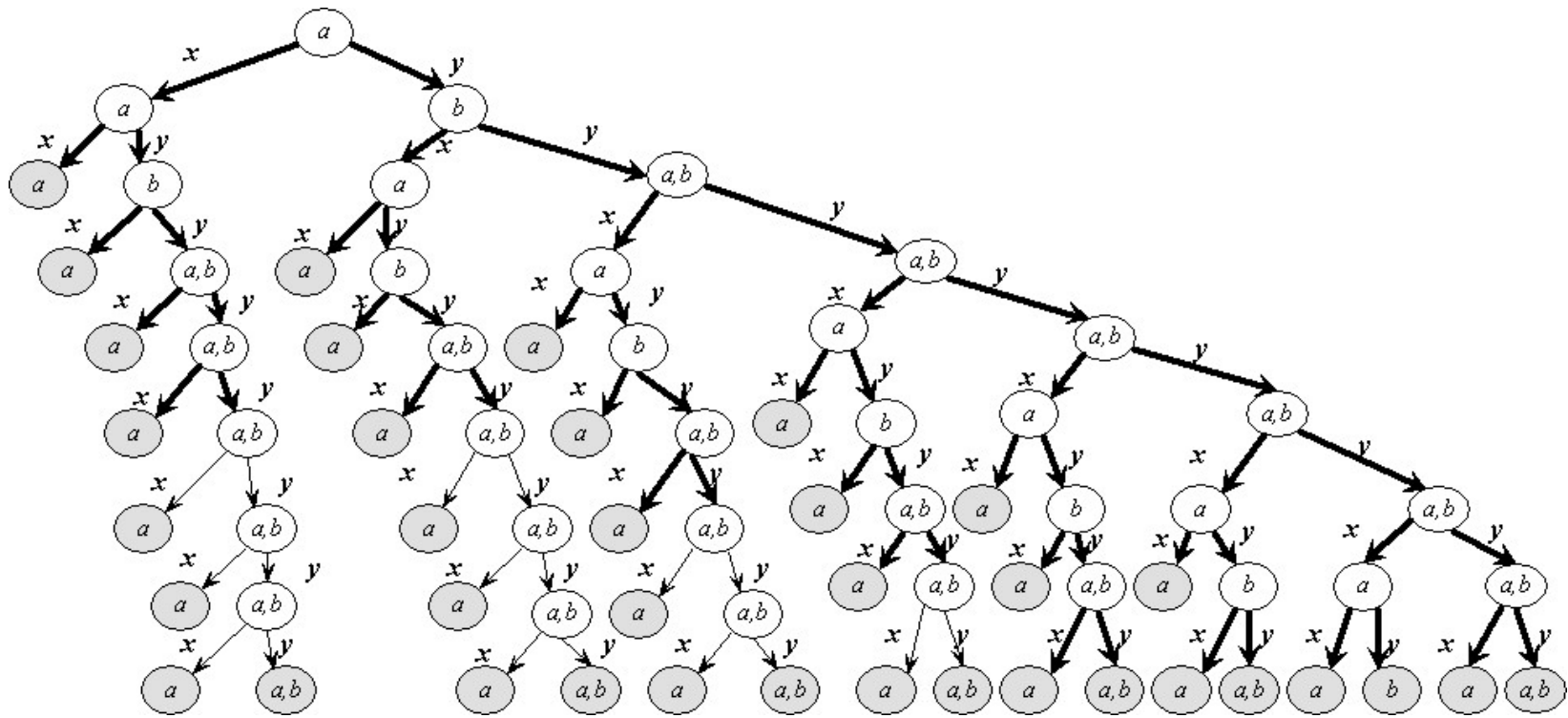
- $K = \{1,2\} \cup \{3,4\}$

The number of different non-empty subsets of  $\{1,2\} \times M = 2^4 - 1 = 15$

- The only sets which do not include non-empty subsets of  $\{1,2\} \times M$  and  $\{3,4\} \times M$  are subsets of  $\{5\} \times M$



# EXAMPLE (2)





# EXPERIMENTAL RESULTS

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- On average, tests w.r.t. a mutation machine *MM* using modified algorithm are twice shorter and this gain increases when *MM* is more deterministic
- On average, tests w.r.t. a “black box” are 1.5 times shorter

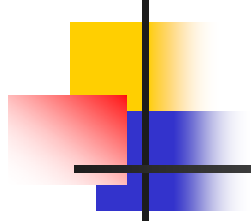
More rigorous analysis is needed to shorten tests w.r.t. a “black box”



# FUTURE WORK

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- Both algorithms do not return a shortest test suite w.r.t.  $\langle S, \sim, Sub_{nd}(MM) \rangle$  and w.r.t.  $\langle S, \sim, \mathcal{R}_m \rangle$
- Adaptive tests using the  $r$ -distinguishability relation could be shorter
- More rigorous experimental results could be interesting



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**Thanks for your attention!**